The Effect of Management Discretion on Hedging and Fair Valuation of Participating Policies with Maturity Guarantees

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Market-consistent Valuation of Insurance Contracts
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Outline

- What are participating policies?
- The challenges faced by with-profits insurers in the U.K.
- Solutions used in practice and suggested in the literature
- Our suggested approach
- Extensions and numerical example
- Conclusions
Participating policies are life-insurance policies that enable policyholders to share in the profits of the insurer. Participating policies take many different forms throughout the world. We will focus on unitised with-profits (UWP) of the following form:

- single premium
- guaranteed growth rate $\gamma$
- regular bonuses
- initially we assume no terminal bonus.
Participating Policies

- Maturity of the contract: $T$
- Portfolio (Insurer’s assets, Reference portfolio): $V(t), t = [0, T]$
- Policyholder’s account: $X(t), t = 0, \ldots, T, X(0) = 1$
- Returns:
  \[ r_V(t) = \log \frac{V(t + 1)}{V(t)}, \]
  \[ r_X(t) = \max \{\gamma, \delta r_V(t)\} \quad \gamma \in \mathbb{R}, \delta \in (0, 1) \]

\[ X(t + 1) = X(t) e^{r_X(t)} = X(t) \max \left\{ e^\gamma, \left(\frac{V(t + 1)}{V(t)}\right)^\delta \right\} \]

- payoff to the policyholder: $X(T)$
Desirable Features of UWP

The policyholder is looking for:

▶ maturity guarantees
▶ regular and sustainable bonuses
▶ transparency
▶ equity exposure

While the insurer’s management wants to maintain their discretion in choosing an investment strategy.
Challenges Faced by UWP Insurers

- Design policies which retain the desirable features of UWP and meet Policyholders Reasonable Expectations.
- Reduce the investment risks faced by the insurer offering the guarantees.
- Calculate Market-consistent fair values.
Recent History

▶ Low inflation, low investment returns
▶ Negative equity returns in recent years
▶ → increasing cost of guarantees

Insurers have responded by using management discretion to:
▶ reduce equity exposure
▶ reduce bonuses
▶ create new products with few or no guarantees

As a result the desirable features of UWP are disappearing.
Question

What other solutions have been suggested?
Wilkie (JIA 1987) and Willder (PhD 2004) suggested the following approach:

- Asset share is given by the value of an investment portfolio, plus put options which match the guarantees.
- Charges are deducted whenever maturity guarantees are increased to pay for the options.
- Charges are deducted/refunded whenever the insurer increases/decreases the riskiness of the investments.
Solution 1

Features

- Allows for management discretion in changes to both asset mix and bonus policy.
- Complex charges.
- Sustainability of bonuses is not certain.
Solution 2 (Risk-neutral Valuation)

Following on from the ideas of Black-Scholes-Merton, the following authors have all used risk-neutral pricing in connection with participating policies:

- Persson and Aase (JRI 1997)
- Grosen and Jorgensen (IME 2000, JRI 2002)
- Miltersen and Hansen (SAJ 2002)
- Tanskanen and Lukkarinen (IME 2003)
- Ballotta (IME 2005)
Solution 2 (Risk-neutral Valuation)

Method

- Model the insurer’s assets as a fixed reference portfolio.
- Fix a bonus distribution mechanism.
- Fair values are determined by risk-neutral pricing under an equivalent martingale measure.
- Set a charging structure so that the policy has an initial fair value of zero.
- Pass the charges to a separate guarantee account which can be used to hedge the investment risks.
Solution 2 (Risk-neutral Valuation)

- Reference Portfolio: $V(t), t = [0, T]$
- Policyholder’s account: $X(t), t = 0, \ldots, T, X(0) = 1$
- Returns:
  \[ r_V(t) = \log \frac{V(t + 1)}{V(t)}, \]
  \[ r_X(t) = \max \{ \gamma, \delta r_V(t) \} \quad \gamma \in \mathbb{R}, \delta \in (0, 1) \]

\[ X(t + 1) = X(t)e^{r_X(t)} = X(t)\max \left\{ e^{\gamma}, \left( \frac{V(t + 1)}{V(t)} \right)^\delta \right\} \]

- Payoff to the policyholder: $X(T)$

**Aim:** Find a replicating portfolio $\tilde{V}$ such that $\tilde{V}(T) = X(T)$
Find a "fair price" for the payoff $X(T)$
Solution 2 (Risk-neutral Valuation)

Features

- Bonuses are transparent and sustainable.
- Asset model is unrealistic - the authors ignore the management’s discretion in changing the asset mix.
Kleinow and Willder (IME 2007) and Kleinow (2006) suggest the following approach:

- Fix a bonus distribution mechanism based on the performance of the insurer’s total assets.
- Determine assets which hedge the guarantees.
- Calculate fair values as the value of the insurer’s portfolio.
- Calculate the premium as the initial fair value.
Solution 2 again

► Reference Portfolio: $V(t), t = [0, T]$
► Policyholder’s account: $X(t), t = 0, \ldots, T, X(0) = 1$
► Returns:

$$r_V(t) = \log \frac{V(t+1)}{V(t)},$$

$$r_X(t) = \max \{\gamma, \delta r_V(t)\} \quad \gamma \in \mathbb{R}, \delta \in (0, 1)$$

►

$$X(t+1) = X(t)e^{r_X(t)} = X(t) \max \left\{ e^\gamma, \left( \frac{V(t+1)}{V(t)} \right)^\delta \right\}$$

► payoff to the policyholder: $X(T)$

**Aim:** Find a replicating portfolio $\tilde{V}$ such that $\tilde{V}(T) = X(T)$
Find a ”fair price” for the payoff $X(T)$

Typically $\tilde{V} \neq V$
Assume that the "reference portfolio" is the actual portfolio of the insurance company?

Consequences:

► The management of the insurer has the right to change the portfolio at any time
► The insurer cannot set up a separate hedge portfolio since this portfolio becomes part of the insurer’s assets
The Contract

- Portfolio of insurance company: \( V(t) \), \( t = [0, T] \)
- Policyholder’s account: \( X(t) \), \( t = 0, \ldots, T \), \( X(0) = 1 \)
- \( X(t + 1) = X(t) e^{rX(t)} \)
  \[ = X(t) \max \left\{ e^{\gamma}, \left( \frac{V(t+1)}{V(t)} \right)^{\delta} \right\} \]

**Aim:** Choose \( V \) such that \( V(T) = X(T) \)
Find a ”fair price” for the payoff \( X(T) \)

\( V \) serves simultaneously as underlying portfolio and hedge portfolio.
The Financial Market Model

- probability space \((\Omega, \mathcal{F}, P)\) and a filtration \(\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}\)
- tradable assets:
  - ZCB with maturity \(T\): \(P(t) \in \mathcal{F}_t, P(T) = 1\)
  - bank account: \(B(t) \in \mathcal{F}_t, B(0) = 1, B(t) \geq B(s)\) for all \(s \leq t\)
  - discount factor \(D(s, t) = B(s)/B(t)\) \(0 \leq s \leq t \leq T\)
  - other assets: \(S_1, \ldots, S_d\) are semimartingales wrt. \(\mathbb{F}\)
The Financial Market Model

- Portfolio strategy: \( a(t), b(t), c(t) = (c_1(t), \ldots, c_d(t)) \) and

\[
V(t) = a(t)P(t) + b(t)B(t) + \sum_{i=1}^{d} c_i(t)S_i(t) \in \mathcal{F}_t
\]

at any \( t = [0, T] \)

- No Arbitrage: \( \exists \ Q \sim P \) such that for \( 0 \leq s < t \leq T \)
  - \( P(s) = \mathbb{E}_Q[D(s, t)P(t) | \mathcal{F}_s] \)
  - \( V(s) = \mathbb{E}_Q[D(s, t)V(t) | \mathcal{F}_s] \) if \( V \) is self-financing

**Aim:** Choose \( (a, b, c) \) such that \( V(T) = X(T) \)
Find a fair price for the payoff \( X(T) \)
Consider the period \((T - 1, T]\)

**Lemma**

\[ V(T) = X(T) \] if and only if the value of the portfolio at time \(T - 1\) is given by

\[ V(T - 1) = a(T - 1)P(T - 1) \]

and

\[ a(T - 1) = \max \left\{ e^\gamma, P(T - 1)^{-\delta} \right\} X(T - 1) \]

is the number of zero-coupon bonds the insurance company buys at time \(T - 1\).

\( V(T - 1) \) is the fair price of \(X(T)\) at time \(T - 1\).
Proof

Conditionally on $\mathcal{F}_{T-1}$:

$$X(T) = \begin{cases} 
e^\gamma X(T-1) & \text{for } \frac{V(T)}{V(T-1)} < e^{\gamma/\delta} \\ 
\frac{X(T-1)}{V(T-1)^\delta} V(T)^\delta & \text{for } \frac{V(T)}{V(T-1)} \geq e^{\gamma/\delta} \end{cases}$$

Define

$$h(y, v) = \begin{cases} 
v^{-1}e^\gamma X(T-1) & \text{for } v < e^{\gamma/\delta} y \\ 
\frac{X(T-1)}{y^\delta} v^{\delta-1} & \text{for } v \geq e^{\gamma/\delta} y \end{cases}$$

$$\implies \frac{X(T)}{V(T)} = h(V(T-1), V(T))$$
Proof

For \( y \in (0, \infty) \):

\( h(y, v) \) is continuous in \( v \) and

\[ \frac{\partial}{\partial v} h(y, v) < 0 \quad \forall \quad v \in [0, \infty) \]

\[ \lim_{v \to \infty} h(y, v) = 0 \quad \lim_{v \to 0} h(y, v) = \infty \]

\( \implies h(y, v) = 1 \) has exactly one solution \( v \)

For a given value \( V(T - 1), V(T) \) must be a non-random number.

\( \implies V(T) = a(T - 1)P(T) = a(T - 1) \)

\[ V(T - 1) = X(T - 1) \max \left\{ e^\gamma, P(T - 1)^{-\delta} \right\} P(T - 1) \]
Remarks

- \( V(T) = a(T - 1)P(T) = a(T - 1) \in \mathcal{F}_{T-1} \), i.e. the payoff to the policyholder is known at time \( T - 1 \).
- We can rewrite the result of the lemma as

\[
V(T - 1) = X(T - 1)C(T - 1)
\]

with

\[
C(T - 1) = \max \left\{ e^\gamma, P(T - 1)^{-\delta} \right\} P(T - 1)
\]

\( C(T - 1) = C(T - 1, P(T - 1)) \) is the \textbf{fair relative value} of the contract at time \( T - 1 \).
Consider the period \((T - 2, T - 1]\)

Lemma: \(V(T) = X(T)\) iff

\[
V(T - 1) = X(T - 1)C(T - 1)
\]

\[
= X(T - 2) \max \left\{ e^\gamma, \left( \frac{V(T - 1)}{V(T - 2)} \right)^\delta \right\} C(T - 1). 
\]

Given \(\mathcal{F}_{T-2}\), \(V(T - 1)\) and \(C(T - 1)\) are random

\[\implies V(T - 1)\] is a deterministic function of \(C(T - 1)\),

\[
V(T - 1) = f(C(T - 1), X(T - 2), V(T - 2))
\]

with \(f(c, x, v)\) is the solution of

\[
f(c, x, v) = \begin{cases} 
  e^\gamma xc & \text{for } e^\gamma > f(c, x, v)^\delta v^{-\delta} \\
  f(c, x, v)^\delta v^{-\delta} xc & \text{for } e^\gamma \leq f(c, x, v)^\delta v^{-\delta}
\end{cases}
\]
Risk-neutral valuation: Period \((T - 2, T - 1]\)

It is easy to check that

\[
f(c, x, v) = \begin{cases} 
  e^{\gamma x c} & \text{for } c < \frac{v}{x} e^{\gamma/\delta - \gamma} \\
  (v^{-\delta} x c)^{\frac{1}{1-\delta}} & \text{for } c \geq \frac{v}{x} e^{\gamma/\delta - \gamma}
\end{cases}
\]

If \(V(T - 2)\) and \(X(T - 2)\) are given, it follows that \(V(T - 1)\) is a contingent claim with underlying process \(P\).

\[
V(T - 1) = f[C(T - 1, P(T - 1)), X(T - 2), V(T - 2)]
\]

Price of this claim at \(T - 2\):

\[
\mathbb{E}_Q[D(T - 2, T - 1)f[C(T - 1), X(T - 2), V(T - 2)] | \mathcal{F}_{T-2}]
\]

Risk-neutral valuation:

\[
\mathbb{E}_Q[D(T - 2, T - 1)V(T - 1) | \mathcal{F}_{T-2}] = V(T - 2)
\]

Therefore, we obtain \(V(T - 2)\) as the solution of

\[
\mathbb{E}_Q[D(T - 2, T - 1)f(C(T - 1), X(T - 2), v) | \mathcal{F}_{T-2}] = v .
\]
Risk-neutral valuation: Period \((T - 2, T - 1]\)

\[ E_Q[D(T - 2, T - 1)f(C(T - 1), X(T - 2), v) | \mathcal{F}_{T-2}] = v. \]

- A solution exists and for this solution holds that
  \[ V(T - 2) = X(t - 2)C(T - 2, P(T - 2)) \]

  where \(C(T - 2, P(T - 2))\) is a contingent claim maturing at time \(T - 2\) with underlying \(P\).

- \(v = C(T - 2)\) solves the equation
  \[ E_Q[D(T - 2, T - 1)f(C(T - 1), 1, v) | \mathcal{F}_{T-2}] = v. \]

- \(C(T - 2)\) is the fair relative value at time \(T - 2\)
- use backward induction
Theorem

Assume that for all \( t = 0, \ldots, T - 2 \) the joint distribution of \( P(t + 1) \) and \( D(t, t + 1) \) conditionally on \( \mathcal{F}_t \) is absolutely continuous wrt. \( \lambda \) on \((0, 1) \times (0, 1)\) and its density is continuously differentiable with respect to \( P(t) \). Assume that \( C(T - 1, P(T - 1)) \) is given by

\[
C[T - 1, P(T - 1)] = \max \left\{ e^\gamma, P(T - 1)^{-\delta} \right\} P(T - 1) .
\]

Then exists a unique solution \( C(t, P(t)) \) of

\[
E_Q \left[ D(t, t+1)f \left\{ C[t+1, P(t+1)], 1, C[t, P(t)] \right\} \mid \mathcal{F}_t \right] = C[t, P(t)] .
\]

for every \( t \). Furthermore, \( C[t, P(t)] \) is a continuous function of \( P(t) \).
Risk-neutral valuation

We call \( C[t, P(t)] \) the fair relative value of \( X(T) \) at time \( t \).

\( V(t) = X(t)C(t, P(t)) \) is called the fair value of \( X \).

Properties of \( V \):

- Theorem: \( V(t) \) solves for every \( t = 0, \ldots, T - 2 \)

\[
\mathbb{E}_Q \left[ D(t, t+1) f \{ C[t+1, P(t+1)], X(t), V(t) \} \mid \mathcal{F}_t \right] = V(t).
\]

- \( \mathbb{E}_Q \left[ \frac{V(t)}{B(t)} \mid \mathcal{F}_s \right] = \frac{V(s)}{B(s)} \quad \forall 0 \leq s < t \leq T \) (the discounted value process is a \( Q \)-martingale)
Why is \( V \) called "Fair Value"

If at every time \( t \) a contingent claim existed with expiry date \( t + 1 \), underlying \( P \) and payoff-function

\[
f \{ C[t + 1, P(t + 1)], X(t), V(t) \}
\]

with

\[
f(c, x, v) = \begin{cases} 
e^\gamma xc & \text{for } c < \frac{v}{x} e^{\gamma/\delta-\gamma} \\ \left(v^{-\delta} xc\right)^{\frac{1}{1-\delta}} & \text{for } c \geq \frac{v}{x} e^{\gamma/\delta-\gamma} \end{cases}
\]

and \( C(t + 1, P(t + 1)) \) is the solution of

\[
\mathbb{E}_Q \left[ D(t + 1, t + 2) f \{ C[t + 2, P(t + 2)], 1, C[t + 1, P(t + 1)] \} \mid \mathcal{F}_t \right] = C[t + 1, P(t + 1)].
\]

with \( C(t + 2, P(t + 2)) \) is the solution of ... with \( C(T - 1, P(T - 1)) = \max \{ e^\gamma, P(T - 1)^{-\delta} \} P(T - 1) \)
Why is $V$ called ”Fair Value”

then

- $V(t)$ is the value of that claim $f(C(t + 1), X(t), V(t))$ at $t$
- the insurance company can buy this claim for $V(t)$
- $V(t + 1)$, the payoff of this claim at time $t + 1$ is equal to the price of a similar claim at time $t + 1$ with maturity $t + 2$
- the payoff at $t + 1$, $V(t + 1)$, can therefore be reinvested into the claim with maturity $t + 2$
- ...
- the payoff at time $T - 1$ is

$$V(T - 1) = X(T - 1)C(T - 1, P(T - 1))$$

$$= X(T - 1) \max \left\{ e^\gamma, P(T - 1)^{-\delta}\right\} P(T - 1)$$

- the insurance company can invest $V(T - 1)$ to buy $X(T - 1) \max \left\{ e^\gamma, P(T - 1)^{-\delta}\right\}$ ZCBs with maturity $T$
- $V(T) = X(T)$. 
Hedging (One-Factor Model)

Consider \((\Omega, \mathcal{F}, \mathbb{P})\) and a filtration \(\mathbb{F} = (\mathcal{F}_t)_{t \in [0, T]}\) generated by a Brownian motion \(W\).

Short interest rate process:

\[
    dr(t) = m_r(t) dt + \sigma_r(t) dW(t)
\]

Bond price process (ZCB with maturity \(T\)):

\[
    dP(t) = P(t) [m_P(t) dt + \sigma_P(t) dW(t)]
\]

Bank account:

\[
    B(0) = 1 \quad dB(t) = r(t) B(T) dt \quad B(t) = \exp \left[ \int_0^t r(s) ds \right]
\]

\(P(t), r(t)\) are adapted to \(\mathbb{F}\), \(B(t)\) is predictable.
Hedging (One-Factor Model)

Risk premium:

\[ \rho(t) = \frac{m_P(t) - r(t)}{\sigma_P(t)} \]

Define \( Q \sim P \) by

\[
\frac{dQ}{dP} = \exp \left( -\int_0^T \rho(s) dW(s) - \frac{1}{2} \int_0^T \rho(s)^2 ds \right)
\]
**Lemma**

Assume $V(t)$ solves for every integer $t = 0, \ldots, T - 2$ the equation

$$EQ[D(t, t + 1)f(C(t + 1, P(t + 1)), X(t), V(t)) \mid \mathcal{F}_t] = V(t).$$

Then there exist predictable processes $a(t), b(t)$ with

$$a(t)P(t) + b(t)B(t) = V(t) \quad \forall \ t \in [0, T],$$

$$V(t) = V(0) + \int_0^t a(t)dP(t) + \int_0^t b(t)dB(t) \quad \forall \ t \in [0, T]$$

and

$$V(T) = X(T)$$
Possible Extensions

- Including some additional options (Guaranteed Annuity Options)
- Different Bonus distribution mechanisms
- Equity exposure
Equity exposure (Binomial Model)

- We know that the insurer would not invest into equity if a perfect hedge is desired.
- We now assume that the insurer wants to setup a different portfolio $\tilde{V}$ with $\tilde{V}(T) \geq X(T)$ where the return of $X$ now depends on $\tilde{V}$.
- It follows immediately that $\tilde{V}(0) \geq V(0)$.
- Assume the policyholder pays $\tilde{V}(0) > V(0)$.
- In this case the insurer should declare a terminal bonus. Otherwise an arbitrage opportunity exists.
  Payoff to the policyholder: $\max\{X(T), V(T)\}$
  If the contract is successfully hedged:
  $\max\{X(T), V(T)\} = V(T)$
- We find that the insurer can invest into equity. We can calculate how much can be invested into equity such that the insurer can at least perfectly hedge at the next time step. The strategy is not unique.
Equity Exposure – Example

Investment opportunities for particular ratios $X(t)/V(t)$. 

T. Kleinow: Participating Insurance Policies
Numerical Example (Binomial Model)

- Guaranteed rate: $\gamma = 0.025$
- Participation rate: $\delta = 0.9$
- Maturity: $T = 4$ years
- Binomial tree for $r(t)$ with $u = 1.5$, $d = 1/u = 2/3$ and $r(0) = 0.035$
- Flat yield curve, $(P(t-1, t+1) = e^{-2r(t)})$

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### Numerical Example (Binomial Model)

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Our Approach

Features

▶ We assume that management exercise their discretion over the choice of the assets to the full in that they choose to perfectly hedge the liabilities.

▶ Note that the hedging portfolio is also the reference portfolio used to determine the bonuses. The reference portfolio is the insurer’s total assets and not some fixed external portfolio.

▶ Fair values allow for management discretion in changing asset mix and future bonuses.

▶ Bonuses are transparent and sustainable.

▶ Only an initial charge is required as the hedging portfolio is self-financing.
Conclusions

We have considered a contract which:

▶ ensures the sustainability of bonuses
▶ has bonuses based on the success of the insurer’s own investments.

We have shown that a perfect hedging strategy:

▶ exists and is unique (in a complete market)
▶ excludes equities

In a Binomial model: We showed how limits can be placed on equity exposure which ensure that assets are at least as large as the guarantees at maturity.

These results can be extended to general financial market models.
Open Questions

- Pricing/Hedging for particular models
- Hedging in incomplete markets
- \( V(T) \geq X(T) \), investment into equity in a general financial market model
- Pricing/Hedging of more complicated contracts (path-dependent bonuses)
- ...

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